

# ANALYSIS OF THE PROPAGATION AND LEAKAGE EFFECTS FOR VARIOUS CLASSES OF TRAVELING-WAVE SOURCES IN THE PRESENCE OF COVERING DIELECTRIC LAYERS

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## Abstract

The fundamental perturbation effects on the propagation properties (phase and leakage constants) of traveling-wave sources due to the presence of covering dielectric layers with arbitrary permittivity and dimensions are investigated in this work. Our analysis is applicable to several well-known and used classes of leaky-wave antennas (slotted rectangular guide, stub-loaded, ridge, stepped, and so forth) when radomes are required for environmental protection. The basic step is given by the quantification of the radiation admittance for the aperture of the coated single-line source via a spectral-domain approach. Through usual transverse resonance techniques, from the straightforward calculation of the phase and leakage constants it is then possible to easily characterize the global radiation properties. Extensions to the case of coated arrays are also carried out. The results show the type and the amount of the modifications that are related to the introduction of radomes in traveling-wave antennas.

## I. Introduction

Environmental protection in many types of radiating structures is usually achieved through suitable dielectric coverings (radomes) [1]. For a wide variety of radiators, such as traveling-wave antennas, this goal can often be accomplished by simply introducing a suitable dielectric layer over the plane of the radiating aperture [2,3]. This introduction of dielectrics acts in a not easily predictable way on the propagation properties of the sources, generally altering both the phase and the leakage constants of the wave.

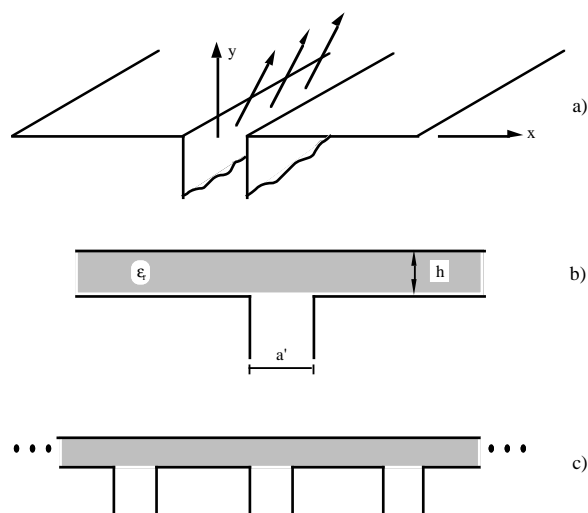
A complete analysis of such perturbation effects due to the presence of dielectric layers is achieved here, based on the characterization of the suitable equivalent-circuit element for the most typical aperture of many leaky-wave structures. This approach is accurate and straightforward, since it allows us to achieve the radiation properties of leaky-wave antennas with radomes by means of simple variations of the radiation admittance in the usual transverse-resonance networks [4]. The procedures are also extended to arrays for bidirectional scanning.

Useful information is thus achieved for the appropriate model and design of radomes in important types of leaky-wave antennas.

## II. Analysis method for single source

As is known, for typical classes of traveling-wave antennas, radiation is achieved through suitable long slots on the metallic walls of the waveguiding structures [4]. In particular, for many different topologies (e.g., leaky-wave antennas based on the rectangular guide, stub-loaded guide, ridge guide, stepped guide, etc. [4,5]), the most convenient practical solution for the single radiating element is usually represented by a slot with a stub terminated in a pair of "wide" shielding baffles, as sketched in Fig. 1a. In this way, the incident field is described prevalently by the leaky wave (usually, the other higher-order excitable modes are suitably attenuated by the stub), which propagates at an angle on the  $yz$  plane and actually radiates in a grounded half-space.

The easiest way for getting the environmental protection of such structures, which is a fundamental practical requirement, is to overlay the aperture with a planar dielectric slab, as shown in Fig. 1b. The perturbation effects due to the introduction of these radomes should be accurately evaluated since the antenna performances can deeply be altered.



**Fig. 1** - a) Typical radiating slotted aperture for leaky-wave antennas; b) the reference aperture with a protecting radome (section); c) aperture for phased arrays with radome (section).

We are therefore interested in deriving a suitable quantification of the radiation for the structure represented in Fig. 1b.

A convenient approach is to characterize the equivalent-circuit element, i.e. the radiation admittance at the aperture section  $y=0$  that takes into account the presence of the dielectric slab (with height  $h$  and relative permittivity  $\epsilon_r$ ). To this aim, we have employed a spectral-domain approach [6], considering the Fourier transform with respect to the directions  $z$  and  $x$  transverse to broadside.

Making use of the equivalence theorem, we can substitute the slot of width  $a'$  with an equivalent magnetic current density  $\mathbf{M} = -\mathbf{y}_0 \times \mathbf{E}$ , which corresponds to a voltage generator in the equivalent transverse transmission line for TE and TM modes with respect to the vertical direction. It is noted that, to our knowledge, the approaches already found in the literature (e.g., [7]) for the case of our interest had considered only a TEM wave incident perpendicularly on the slab, i.e., along  $y$ .

We may assume that the tangential electric field at the aperture is essentially directed along the  $x$  direction, since, as mentioned above, in the lower air-filled stub region of width  $a' < \lambda_0$  we suppose to have only an incident dominant TE mode above cutoff with an almost horizontal electric field. The field is also independent of the  $x$  direction so that the dominant mode can be viewed as a TEM mode traveling at an angle with respect to the  $y$  direction (i.e., on the  $yz$  plane). With this assumption, the equivalent magnetic currents are essentially directed in the longitudinal  $z$  direction ( $\mathbf{M} = M_z \mathbf{z}_0$ ).

Making use of this reasonable approximation, from the equivalent transmission line, we can express the spectral transverse field components as:

$$\begin{cases} \widetilde{\mathbf{E}}_x = M_z \\ \widetilde{\mathbf{H}}_z = - \left( \frac{k_z^2}{k_t^2} Y_{in}^{TE} + \frac{k_x^2}{k_t^2} Y_{in}^{TM} \right) M_z \end{cases} \quad (1a)$$

$$\begin{cases} Y_{in}^{TE} = \frac{k_{ye}}{\omega \mu} \frac{k_{y0} \cos(k_{ye} h) + j k_{ye} \sin(k_{ye} h)}{k_{ye} \cos(k_{ye} h) + j k_{y0} \sin(k_{ye} h)} \\ Y_{in}^{TM} = \frac{\omega \epsilon_0 \epsilon_r}{k_{ye}} \frac{k_{ye} \cos(k_{ye} h) + j \epsilon_r k_{y0} \sin(k_{ye} h)}{\epsilon_r k_{y0} \cos(k_{ye} h) + j k_{ye} \sin(k_{ye} h)} \end{cases} \quad (1b)$$

where  $k_t$  is the transverse wavenumber with respect to the  $y$  direction, given in terms of the horizontal  $k_x$  and longitudinal  $k_z$  wavenumbers:  $k_t^2 = k_x^2 + k_z^2$  (a common dependence  $\exp(-jk_z z)$  for the fields is assumed and suppressed);  $Y_{in}^{TE}$  and  $Y_{in}^{TM}$  are the input admittances of the equivalent transmission line terminated on the characteristic admittances ( $Y_o^{TE}$  and  $Y_o^{TM}$ ), which represent the infinite upper air region ( $k_{y0}$ ,  $k_{ye}$  are the vertical wavenumbers in the air and in the dielectric, respectively).

It is then possible to give a modal expansion of the  $x$ -dependent part of the field on the aperture. Then, from the enforcement of the continuity for  $E_x$  and  $H_z$ , using the orthogonality properties of the harmonic functions, and

after some other analytical manipulations, we may reach a variational expression for the radiation admittance that is stationary with respect to the aperture field  $E_a$ . Therefore, we can approximate the aperture field with a constant value along  $x$ , obtaining the resulting reference expression for the radiation admittance  $Y_R$ :

$$Y_R = \frac{4}{\pi a'} \int_0^\infty \left( \frac{k_z^2}{k_t^2} Y_{in}^{TE} + \frac{k_x^2}{k_t^2} Y_{in}^{TM} \right) \frac{\sin^2(k_x a'/2)}{k_x^2} dk_x \quad (2)$$

which has to be numerically evaluated in the complex  $k_x$  plane.

A careful choice of the integration path allows us to avoid poles, which correspond to the surface-wave modes of the dielectric slab of height  $h$ , and the branch-point singularity in  $k_t = k_0$ .

It is thus possible to quantify the radiation admittance (real and imaginary part) as a function of the width and permittivity of the layer and of the aperture dimension. Relevant results will be discussed in Sect. IV.

### III. Analysis method for phased arrays

The determination of the radiation admittance in the presence of radomes is now completed by considering linear phased arrays of parallel line sources [4,8], as sketched in Fig. 1c.

The mutual coupling between the various sources has been taken into account rigorously with the unit-cell approach [8] (the single cell is limited by suitable phase-shift walls). As is well known, due to periodicity, a set of infinite space harmonics is excited in the upper periodical region, with wavenumbers given by:

$$k_{xnp} = k_{xop} + \frac{2n\pi}{p}, \quad n=0, \pm 1, \dots; \quad k_{xop} = \frac{\Phi_c}{p} \quad (3)$$

where  $\Phi_c$  is the imposed phase shift between successive line sources at distance  $p$ . Other relations are readily established with the air and dielectric  $y$ -wavenumbers ( $k_{ynp}$ ,  $k_{ynp\epsilon}$ ) and the  $x$ -wavenumber in the bottom region  $k_x$ .

The corresponding expressions for the characteristic admittances  $Y_o$ ,  $Y_{onp}$ ,  $Y_{onp\epsilon}$  (in the stub, in the upper air, and in the dielectric regions, respectively) are given by standard formulas [8].

The problem of characterizing the radiation for slotted arrays with dielectrics has been approached in the literature with different methods (see, e.g., [9]). To compute the radiation admittance, we have adapted here an approach given in [10] for another structure. Because the radiating slot is longitudinally continuous, the modes excited in the unit cell are LSE with respect to the  $z$  direction, and all the higher modes in the stub region excitable by the discontinuity can be neglected.

Then the continuity condition for the tangential fields is imposed on the aperture. On the bottom side of the aperture we assume a constant field for any value of the phase shift, expressing the ratio between  $E_x$  and  $H_z$  as  $Y_{uc}$ ,

the admittance that is seen by the dominant mode in the  $y$  direction in the stub region (this approximation is justified when the slot is small with respect to the period  $p$  of the array). On the top side of the aperture, we can express the periodic modes with suitable sum expansions involving the LSE admittances  $Y_{in}$ . From the electric and magnetic field continuity, making use of the orthogonality properties of the exponential functions, after other analytical manipulations we can find the expression for the unit-cell admittance  $Y_{uc}$ , normalized with respect to the characteristic admittance of the stub region:

$$\begin{aligned} \frac{Y_{uc}}{Y_o} &= \frac{a'}{p} \sum_{n=-\infty}^{\infty} \frac{Y_{in}}{Y_o} \frac{\sin(k_{xnp}a'/2)}{k_{xnp}a'/2} \\ \frac{Y_{in}}{Y_o} &= \frac{Y_{onpe}}{Y_o} \frac{Y_{onpe}/Y_{onpe} + j \tan(k_{ynpe}h)}{1 + j (Y_{onpe}/Y_{onpe}) \tan(k_{ynpe}h)} \end{aligned} \quad (4)$$

The complete characterization of the radome effects on the radiation admittance allows us to achieve the basic necessary information on the radiation features of leaky-wave antennas, as presented in the next section.

#### IV. Results and discussion

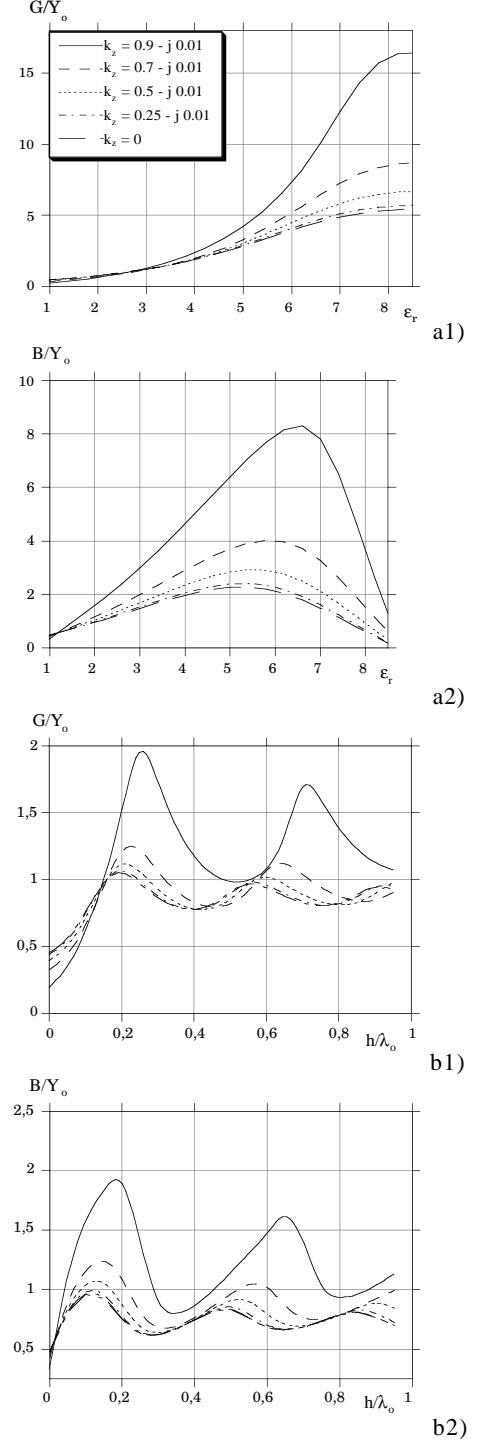
For the single element (Fig. 1b), it is first significant to quantify how the parameters of the coating layers can affect the radiation admittance, in comparison with the case of the absence of dielectric (where such admittance may be evaluated with quite accurate simple analytical forms [8]).

In Fig. 2, a parametric analysis is presented for the radiation admittance (normalized real and imaginary parts) as a function of the involved physical quantities (radome permittivity and height, and complex longitudinal wavenumber), according to the numerical evaluations derivable from Eq. (2).

The results presented here give an immediate comprehension of the kind and the amount of the alterations induced by radomes on the radiation admittance. Furthermore, such a quantification permits a correct analysis of the radiative properties of any topology of leaky-wave antennas by applying standard transverse-resonance network analyses [4,7]. From the achievement of the radiation admittance it is thus possible to achieve simply the phase and the leakage constants (and thus the radiative features) for all the usual types of radiators presenting the aperture as in Fig. 1b.

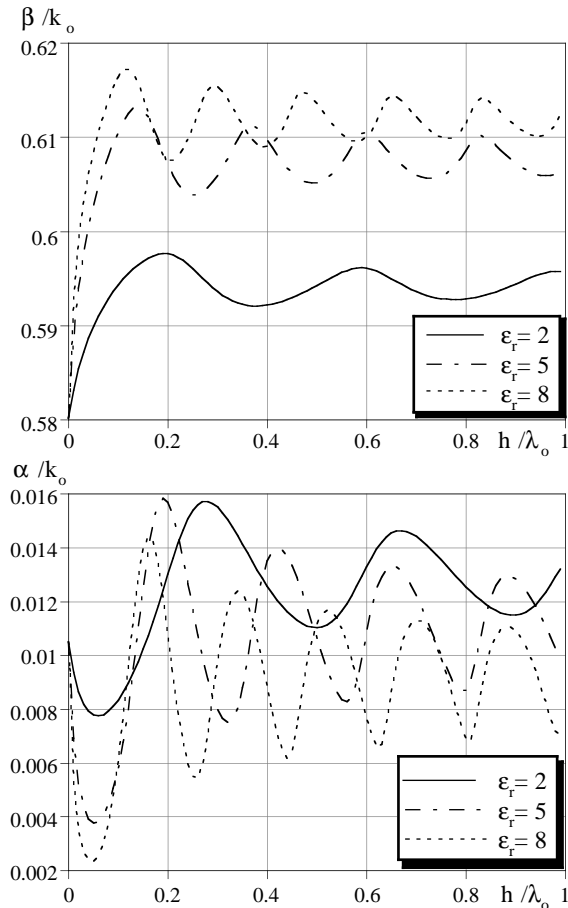
Due to space limits, we can show here just some representative results for a recent interesting geometry, based on a “stepped” rectangular-guide leaky-wave antenna [2,5]. By using a standard equivalent network solved with the transverse-resonance technique, it has been possible to evaluate, in an easy and accurate way, the phase and leakage constants of this structure as a function of the involved physical parameters, without and with radomes.

Referring to a single stepped antenna for a fairly common choice of the parameters [5], we show in Fig. 3 some behaviors of the normalized phase (up) and leakage (down) constants, calculated vs. the radome normalized height, as the dielectric permittivity is varied.



**Fig. 2** - Behaviors of the radiation admittance  $Y_R = G + jB$ , normalized with respect to the incident-mode admittance  $Y_o$ , for the structure of Fig. 1b as a function of the parameters of the radome:  $G/Y_o$  (a1) and  $B/Y_o$  (a2) as the permittivity varies ( $f = 15$  GHz,  $a' = 3$  mm;  $h = 0.1 \lambda_o$ );  $G/Y_o$  (b1) and  $B/Y_o$  (b2) as the normalized height varies ( $f = 15$  GHz,  $a' = 3$  mm;  $\epsilon_r = 2$ ). The admittance quantities have been calculated for different values of the longitudinal wave-number  $k_z$  (indicated in the legend).

The results of this analysis show that the dielectric covering significantly modifies both the phase and the leakage constants of the radiated wave in peculiar fashions. A modification of the expected radiation pattern is thus visible as concerns with both a shifting of the pointing direction and a variation of the angular width of the beam. The amount of such alterations are anyway deeply dependent on the choice of the radome parameters (permittivity and width), as results from Fig. 3.



**Fig. 3** - Effects of radomes on a single-element stepped leaky-wave antenna: variations of the normalized phase  $\beta/k_0$  (up) and leakage  $\alpha/k_0$  (down) constants vs. height (in terms of free-space wavelengths  $\lambda_0$ ) for different values of permittivity  $\epsilon_r$ . Parameters (see [5] for the meaning of the relevant physical quantities):  $a=12.954$  mm,  $s=a/2$ ,  $l=a/4$ ,  $a'=2.5$  mm,  $c=2$  mm,  $d=3.5$  mm,  $b_l=5.8$  mm,  $b_r=1.2$  mm,  $f=15$  GHz.

Interesting behaviors have been found for arrays too, according to the procedure outlined above. In this case, it has been seen that in particular the leakage amount (calculable vs. the phase shift between the elements, again as permittivity and height are varied) may result deeply altered. Also undesirable effects such as blind spots may be found. The limited space here does not allow us to discuss further results and the relevant physical significance in detail.

## V. Conclusion

A straightforward and complete method has been presented to evaluate the effects of radomes in significant classes of traveling-wave antennas. Both for single elements and for linear arrays, a characterization of the radiation admittance for typical apertures have been achieved as a function of the physical quantities involved. On this basis, the evaluation of the radiation alterations due to the presence of dielectric layers appears immediate by calculating the phase and leakage constants of any structure, via suitable transverse-resonance approaches for the equivalent networks.

Representative data have been derived also for some reference structures of leaky-wave antennas. The basic results of our parametric analysis show that both the phase and the leakage amounts are significantly modulated, according to peculiar patterns, by the radome height and permittivity. Thus, sensitive variations of the direction and of the width of the beam can occur. Deep modifications may be found in the case of coated phased arrays as well.

Such information should be carefully taken into account when coverings are introduced on these radiators. The synthesis techniques to design radomes in the most typical leaky-wave antennas appear now possible with rather simple procedures and reduced computational efforts.

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